

Year 12 Mathematics Applications
Test 6 2020

Networks and Decision Mathematics
Calculator Free

STUDENT'S NAME

Solutions

DATE: Thursday 10th September

TIME: 50 minutes

MARKS: 50

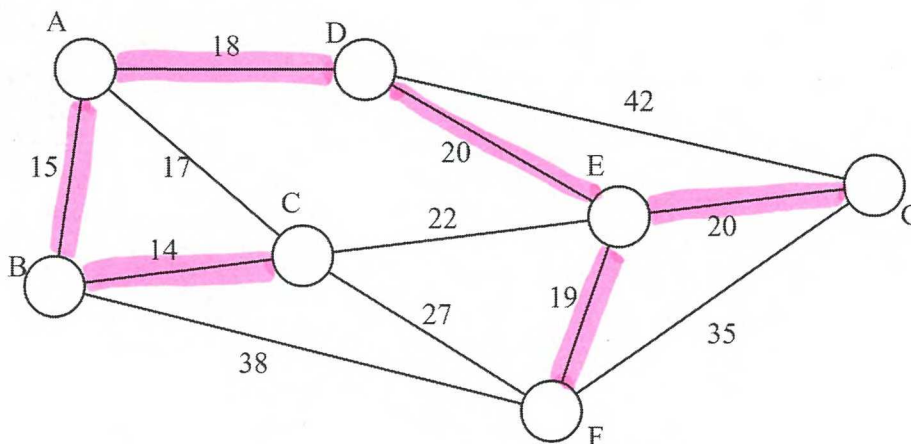
INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

1. (5 marks)

Consider the network diagram below, which shows the distances between various towns in kilometres.



The towns are to be connected by a system of cables laid underground along some of the roads.

- (a) State the minimum total length of cable required to connect each town to the network and clearly highlight the spanning tree on the diagram above. [3]

$$15 + 14 + 18 + 20 + 19 + 20 = 106 \text{ km}$$

✓ 3 correct paths
 ✓ all correct and indicated on graph
 ✓ sums scores

- (b) State the route and length of the shortest path from town A to town G. [2]

$$A - D - E - G = 58 \text{ km}$$

2. (8 marks)

The time, in minutes, to walk between 7 landmarks in King's Park are listed in the table below.

	A	B	C	D	E	F	G
A	-	17	21	27	-	18	-
B	17	-	32	19	29	-	24
C	21	32	-	-	32	25	20
D	27	19	-	-	29	-	13
E	-	29	32	29	-	14	26
F	18	-	25	-	14	-	33
G	-	24	20	13	26	33	-

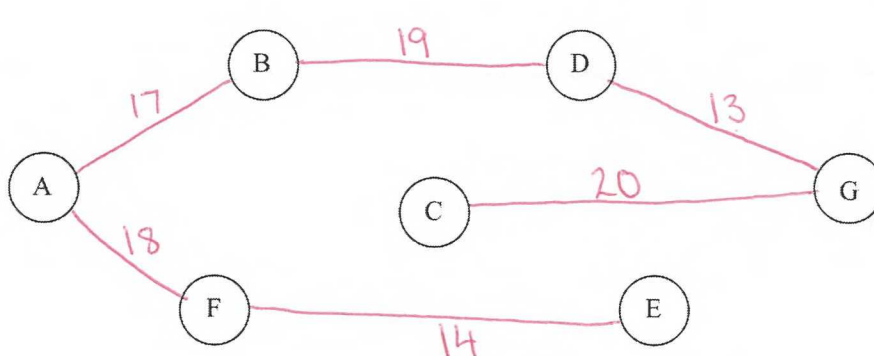
The groundskeepers at King's Park need to upgrade the network of paths around the park. In order to do this they close as many of the paths as possible, leaving a network of paths open that creates a *minimum spanning tree*.

(a) Using Prim's Algorithm or otherwise, determine the total time of the minimum spanning tree. [4]

$$17 + 18 + 19 + 13 + 14 + 20 = 101 \text{ mins}$$

- ✓ 2 correct connections
- ✓ 3 correct
- ✓ correct spanning tree
- ✓ sums values

(b) Clearly indicate the minimum spanning tree found in (a) on the diagram below. [2]



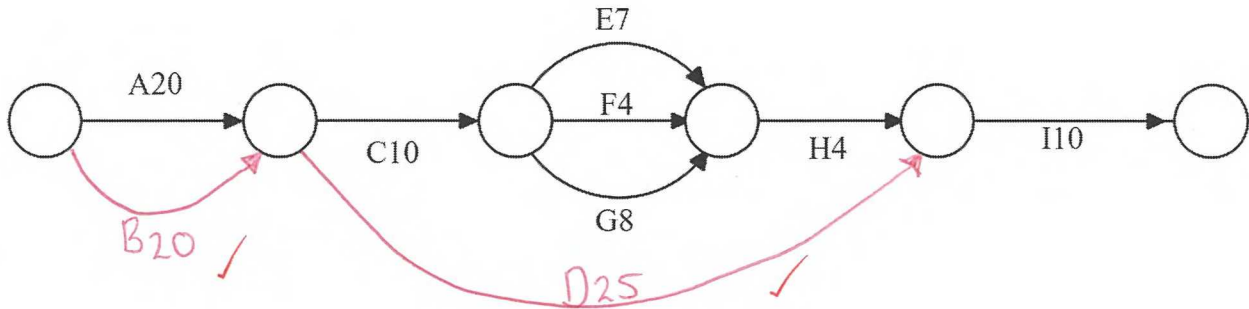
- ✓ correct tree
- ✓ weights

(c) Given that it is not possible to walk directly from landmark B to landmark C, how much longer is the journey from B to C? [2]

was 32 Now 52
 \therefore 20 mins longer

3. (9 marks)

Daniel is setting up a hall for a birthday party. The diagram below shows the project network for this task. Each activity can only be completed by one person and the associated time is measured in minutes.



(a) Use the following table to complete the above network by adding any missing tasks to the diagram. [2]

Task	Time	Immediate Predecessors
A	20	-
B	20	-
C	10	A,B
D	25	A,B
E	7	C
F	4	C
G	8	C
H	4	E,F,G
I	10	D,H

(b) Determine the minimum completion time for this project and state the critical path(s). [3]

A - D - I ✓
 OR
 B - D - I ✓
 55 mins ✓

(c) What is the earliest start time for task H? [1]

38 mins

(d) What is the latest start time for task F? [1]

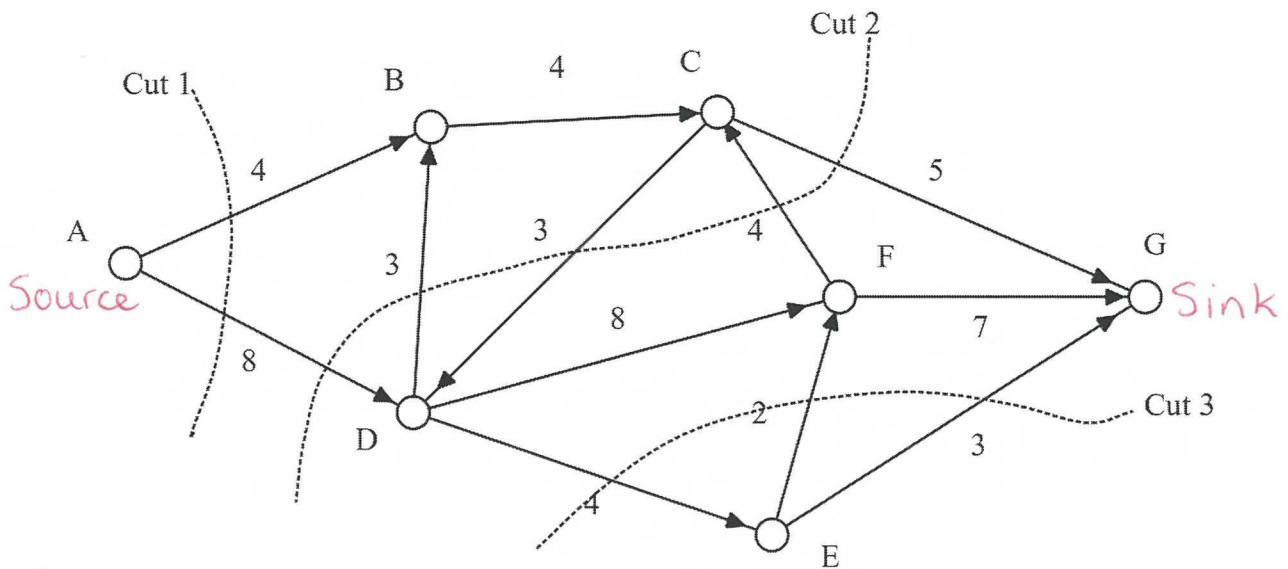
37 mins

(e) How much float time is there for task C? Explain what this means. [2]

3 mins. Task can be delayed 3 mins without affecting completion time.

4. (6 marks)

Consider the network below



(a) Clearly label the **source** and the **sink** [1]

(b) What is the value of Cut 1? [1]

12

(c) Adam claims that the value of Cut 2 is 23, Ben claims it's value is 20 and Chris claims it's value is 16. Who is correct? Explain your answer. [2]

Chris is correct

C-G, C-D and A-D count

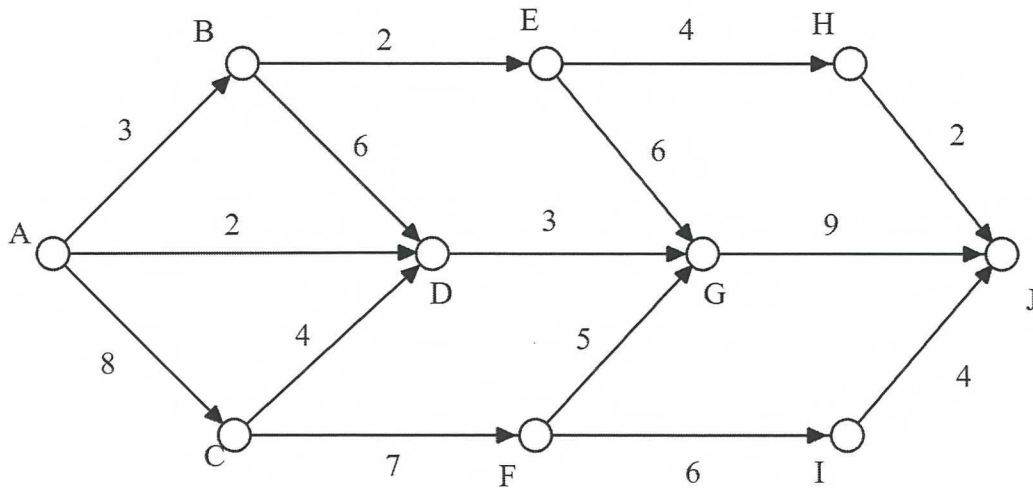
F-C, D-B don't as they travel from sink to source.

(d) Is Cut 3 valid? Explain your answer. [2]

No, does not separate source & sink.

5. (6 marks)

The diagram below represents a rail network linking zones in A to J in a city. The number on each edge represents the maximum number of passengers per minute that can travel along that link during peak periods (in hundreds).



- (a) By listing the different paths and their flow rate, determine the maximum number of passengers per minute that can travel from A to J during peak periods. [4]

$A B E H J = 200$
 $A B D G J = 100$
 $A D G J = 200$
 $A C F G J = 500$
 $A C F I J = 200$
1200 passengers per minute

✓ 2 correct paths & flow
 ✓ All correct paths & flow
 ✓ sums
 ✓ hundreds

- (b) What effect, if any, would there be on the maximum flow of passengers from A to J if the capacity of link F to G was increased by 200 passengers per minute. Justify your answer. [2]

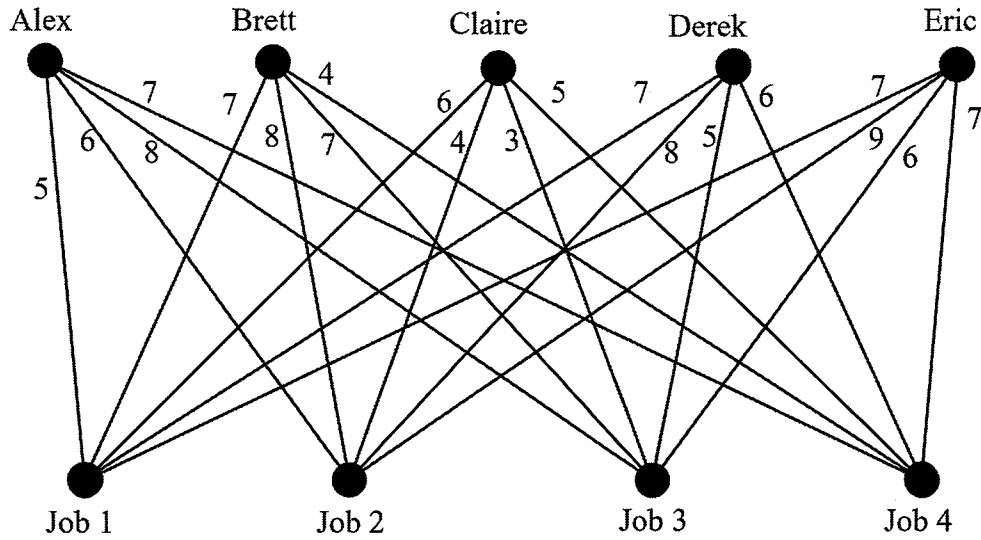
No effect

ACFGJ increases by 1

ACFIJ decreases by 1

6. (7 marks)

A factory has five workers, Alex, Brett, Claire, Derek, and Eric and four jobs to complete. The time in hours, each worker can complete a particular job is given in the weighted bipartite graph below.



(a) Complete the matrix associated with the bipartite graph above [1]

	Job			
	1	2	3	4
Alex	5	6	8	7
Brett	7	8	7	4
Claire	6	4	3	5
Derek	7	8	5	6
Eric	7	9	6	7

(b) Using the Hungarian algorithm on the following page, determine which job should be allocated to each worker so that the total time is minimised. [5]

$$\begin{bmatrix} 5 & 6 & 8 & 7 & 0 \\ 7 & 8 & 7 & 4 & 0 \\ 6 & 4 & 3 & 5 & 0 \\ 7 & 8 & 5 & 6 & 0 \\ 7 & 9 & 6 & 7 & 0 \end{bmatrix}$$

Add 0 column ✓

$$\begin{bmatrix} 0 & 2 & 5 & 3 & 0 \\ 2 & 4 & 4 & 0 & 0 \\ \hline 1 & 0 & 0 & 1 & 0 \\ 2 & 4 & 2 & 2 & 0 \\ 2 & 5 & 3 & 3 & 0 \end{bmatrix}$$

Column reduction ✓

Lines ✓

$n \neq 5$

subtract 2 from uncovered
& add to intersections ✓

$$\begin{bmatrix} 0 & 0 & 3 & 3 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ \hline 3 & 0 & 0 & 3 & 2 \\ 2 & 2 & 0 & 2 & 0 \\ 2 & 3 & 1 & 3 & 0 \end{bmatrix}$$

$n = 5$

Worker	A	B	C	D	E
Job	1	4	2	3	0

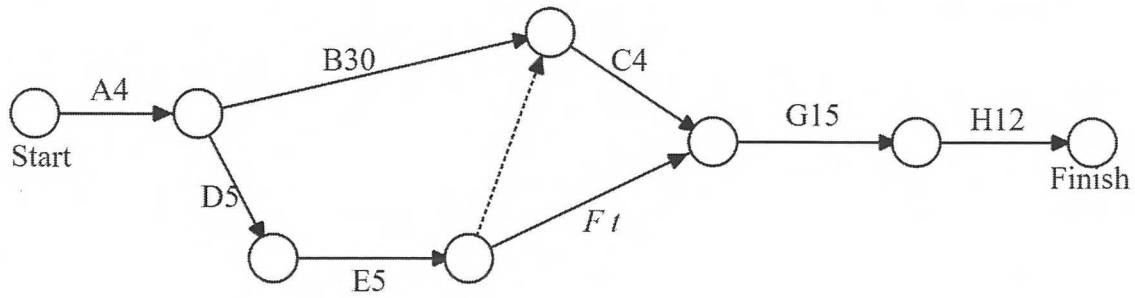
(c) State the total time taken to complete the four jobs

[1]

18 hrs

7. (9 marks)

The network below represents the various activities involved in assembling a computer.



(a) Use the network above to complete the table below, which describes the activities involved in this project. [2]

Activity	Description	Immediate Predecessor	Time (minutes)
A	Install motherboard	—	4
B	Test hard drive	A	30
C	Install hard drive	B, E	4
D	Install I/O ports	A	5
E	Install DVD-RW	D	5
F	Test DVD-RW	E	t
G	Install operating system	C, F	15
H	Test assembled computer	G	12

(b) Explain the purpose of the dotted line on the network. [2]

✓ mention predecessor! It shows that task C cannot commence until both task B and E are complete. However task F only requires E to be complete.
 ✓ context of question
 (Dummy = 0 marks)

(a) The minimum completion time for this project is exactly 65 minutes. State a possible critical path. [2]

A B C G H

✓ evidence of forward or Backwards scan

✓ Correct path

(b) Given that F is not on the critical path, determine the possible values of t . Justify your answer. [3]

EST for F = 14

✓ 24

Must be complete by 38

✓ 0 & correct Boundaries ($0 < t < 24$)

∴ $0 < t < 24$

✓ Justification